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Some Comments on Recent Tevatron Luminosity Performance

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The recent high-luminosity operation of the Tevatron has led some to question just how well the collider is running relative to its ultimate performance for the given set of operational parameters. To get a sense of where things reside, we consider below a "perfect store" – one in which every particle participates in the integrated luminosity, and where the luminosity lifetime is governed solely by the rate at which particles are "consumed" through collisions. From this, we can estimate the integrated luminosity per week for such an ideal condition, or for when stores are intentionally ended early, and compare these results with current Tevatron operations.

To begin, we note that a "perfect store," given enough time, would deliver an integrated luminosity I_0 equal to the number of particles "consumed" divided by the interaction cross section; the luminosity delivered to each experiment would be this number divided by the number of experiments. For a collider with equal number of bunches in each beam, and equal bunch populations, the ultimate integrated luminosity for the store delivered to each experiment would be

$$I_0 = \frac{N_{total}}{n\Sigma} = \frac{B\ N}{n\Sigma}, \tag{1}$$

where B is the number of bunches per beam, N is the number of particles per bunch, n is the number of interaction points, and Σ is the interaction cross section.

To arrive at this conclusion analytically, consider the ideal case of a collider with equal bunch populations in each beam, and where the beam size at the interaction point and the bunch length do not change during the store. The luminosity can be written as

$$\mathcal{L} = \frac{f_0 B N^2}{4\pi\sigma^{*2}} \cdot \mathcal{H}. \tag{2}$$

Here, σ^* is the transverse beam size (considered to be round) at the interaction point, f_0 is the revolution frequency, and $\mathcal{H}$ is the hour glass form factor. Furthermore, suppose the rate at which the particles in each beam leave the accelerator is given solely by the particle interaction rate, namely,

$$B\dot{N} = -\mathcal{L}\ \Sigma\ n. \tag{3}$$

Inserting Eq. 2 into Eq. 3 and integrating leads to

$$\mathcal{L}(t) = \frac{\mathcal{L}_0}{\left[1 + \left(\frac{n\mathcal{L}_0\Sigma}{BN_0}\right) t\right]^2}$$

where N_0 and $\mathcal{L}_0$ are the initial bunch intensity and initial luminosity at time $t = 0$.

The integrated luminosity from the beginning of the store until time $t = T$ is then

$$I \equiv \int_0^T \mathcal{L}(t)dt = \frac{\mathcal{L}_0 T}{1 + \mathcal{L}_0 T(n\Sigma/BN_0)}. \tag{4}$$

Thus, assuming the store ends intentionally at time $T >> BN_0/(n\mathcal{L}_0\Sigma)$, the asymptotic integrated luminosity of the store will be

$$I \longrightarrow \frac{BN_0}{n\Sigma};$$

that is, the maximum total integrated luminosity is the total number of particles in each beam lost at each interaction point, divided by the interaction cross section.

In the case of the Tevatron, the population of antiprotons per bunch, N_2, say, is less than that of the protons, N_1, and so the luminosity is

$$\mathcal{L} = \frac{f_0 B N_1 N_2}{4\pi\sigma^{*2}} \cdot \mathcal{H}. \tag{5}$$

Assuming a one-to-one correspondence in the rate at which protons and antiprotons are consumed, we define $N_2(t) = N(t)$, $N_1(t) = N(t) + N_r$, where, $N_r = N_1^0 - N_2^0$, and

$$B\dot{N} = -\mathcal{L}\;\Sigma\;n.$$

Here, N_1^0 and N_2^0 are the initial bunch intensities of each species at the beginning of the store. Again, substituting Eq. 5 along with the definitions of $N_1(t)$ and $N_2(t)$ into the above differential equation and integrating, we get

$$N(t) = \frac{N_2^0\;N_r}{N_1^0 e^{N_r C t} - N_2^0} \tag{6}$$

where $C \equiv n\mathcal{L}_0\Sigma/BN_1^0N_2^0 = nf_0\mathcal{H}\Sigma/4\pi\sigma^{*2}$. Thus, the luminosity evolves with time according to

$$\mathcal{L}(t) = \mathcal{L}_0\;\frac{N_r^2 e^{N_r C t}}{(N_1^0 e^{N_r C t} - N_2^0)^2}$$

which at large t becomes

$$\mathcal{L}(t) \sim \mathcal{L}_0 \left(1 - \frac{N_2^0}{N_1^0}\right)^2 e^{-N_r C t}.$$

The integrated luminosity, over a time period T, is then

$$I \equiv \int_0^T \mathcal{L}(t)dt = \frac{BN_2^0}{n\Sigma} \cdot \left(\frac{e^{N_r C T} - 1}{e^{N_r C T} - \frac{N_2^0}{N_1^0}}\right) \quad \Longrightarrow \quad I_0 = \frac{BN_2^0}{n\Sigma}, \;\text{ as } t \to \infty. \tag{7}$$

As expected, the resulting maximum integrated luminosity is similar to our last result, where here the total number of particles used is that of the less intense beam.

Let's ask the question(s), If the Tevatron operated under these ultimate conditions, how long would a typical store last, how many stores would occur per week, and thus what would be the integrated luminosity per week? Let's assume that a store lasts until a fraction f of the possible integrated luminosity is obtained. Furthermore, assume a turn-around time, T_a, between stores. Using the result in Eq. 7, the length of each store would be

$$T_f = \frac{1}{N_r C} \ln\left(\frac{1 - fN_2^0/N_1^0}{1-f}\right) \tag{8}$$

at which time the luminosity would have been reduced by a factor of

$$\mathcal{L}(T_f)/\mathcal{L}_0 = (1-f)(1-fN_2^0/N_1^0). \tag{9}$$

The number of stores per week would be $N_{stores} = 168$ hours$/(T_f + T_a)$, and the integrated luminosity per week would be $I_{week} = N_{stores} \cdot f \cdot I_0$.

Take parameters similar to today's Tevatron operation. The Tevatron, with an average radius of 1 km, has 36 bunches in each counter-rotating beam, and a transverse beam size of 25 μm at each collision point; the factor $\mathcal{H} \approx 0.65$. Our results assume these parameters are fixed.

Take $N_1^0 = 2.7 \times 10^{11}$ for the proton beam, and $N_2^0 = 7 \times 10^{10}$ for the antiproton beam. Let's take a cross section of $\Sigma = 60$ mb. Suppose an average store lasts long enough to yield 85% of the maximum I_0 for our example and that the turn-around time is $T_a = 2$ hours. Note, that the final luminosity would be roughly 10% of the initial luminosity under these conditions. Then for these parameters,

$$\begin{aligned}
\mathcal{L}_0 &\approx 270\ \mu\text{b}^{-1}/\text{sec}, \\
T_f = T_{0.85} &\approx 48\ \text{hr}, \\
I_0 &\approx 21\ \text{pb}^{-1}/\text{store}, \\
I_f = I_{0.85} &\approx 18\ \text{pb}^{-1}/\text{store}, \\
N_{stores} &\approx 3.3, \\
I_{week} &\approx 60\ \text{pb}^{-1}.
\end{aligned}$$

Plots of instantaneous and integrated luminosity for a single store under these conditions are provided in Figure 1.

Recent weeks of Tevatron operation, in which downtime due to failures was not a major factor, have yielded integrated luminosities of 40-50 pb^{-1} per week. (See Figure 2.) In reality, many factors (beam-gas interactions, intra-beam scattering, noise sources, *etc.*) cause the luminosity lifetime to be shorter than that given by our simple analysis above, and thus more stores of shorter duration are generated per week at the Tevatron. Nonetheless, it is very interesting to note that the overall integrated luminosity per week of the Tevatron is fairly consistent with this last result, only 15-30% away from this "ultimate" operation.

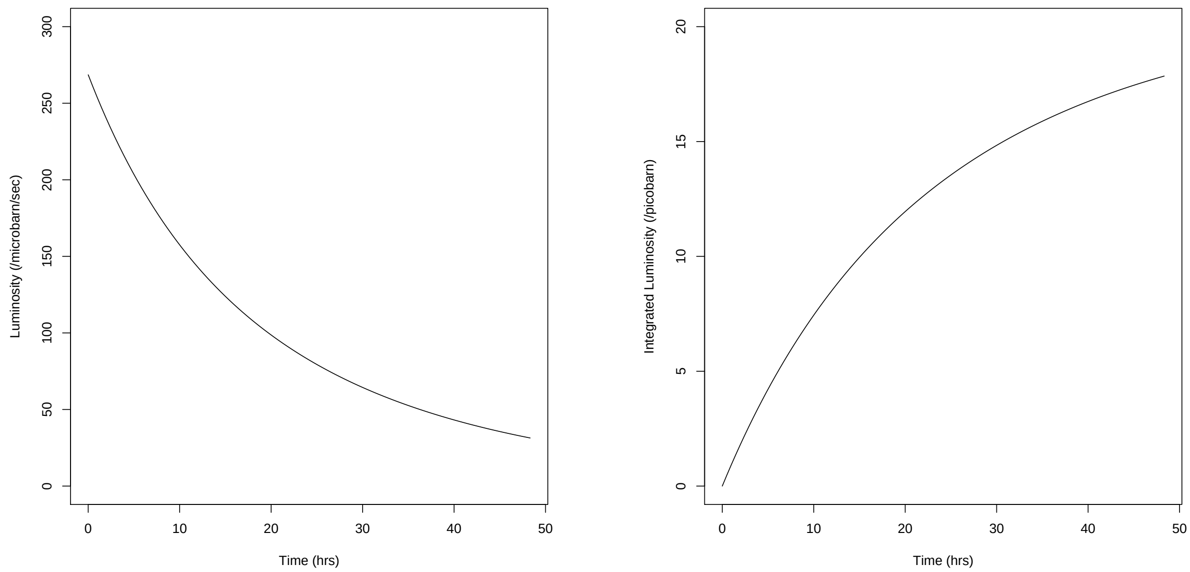

Figure 1: Instantaneous (left) and integrated (right) luminosity *vs.* time through the "perfect" store, using parameters above.

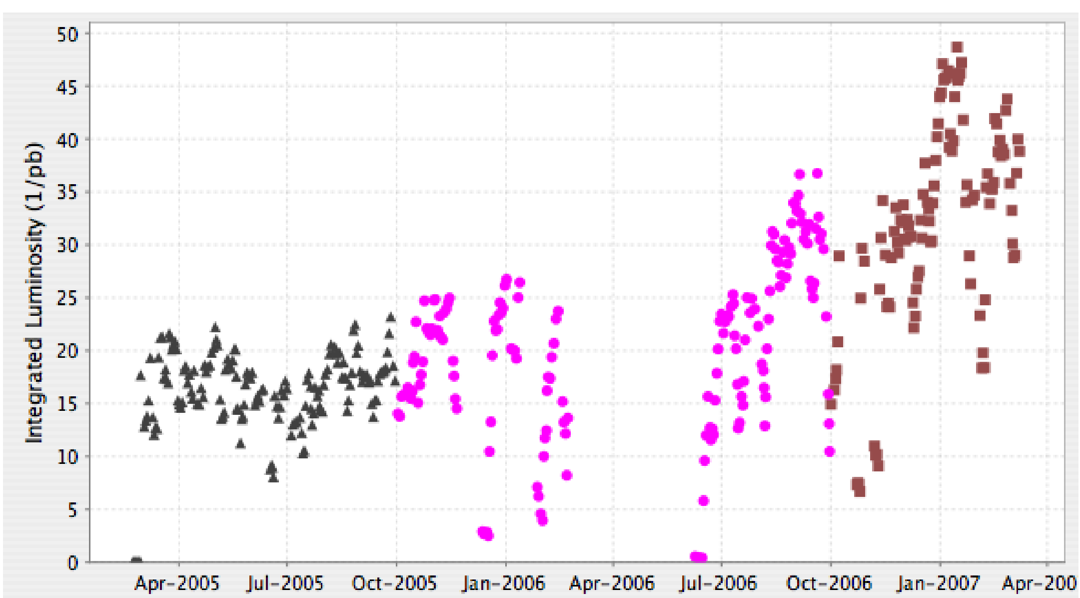

Figure 2: Weekly integrated luminosity in the Tevatron over the span of the past year.

As a follow-on, consider that the fraction f was chosen arbitrarily above to be 85%, as that is essentially the current practice. What value will maximize the weekly integrated luminosity under our assumptions? Waiting long times to grab the last extra bit of events may not be worth the wait. Differentiating the weekly integrated luminosity, we find that the optimal value of f, $\hat{f}$, must satisfy

$$\frac{T_a}{N_r C} + \ln\left(\frac{1 - \hat{f}N_2^0/N_1^0}{1 - \hat{f}}\right) = \frac{\hat{f} \cdot (1 - N_2^0/N_1^0)}{(1 - \hat{f})(1 - \hat{f}N_2^0/N_1^0)}.$$

For the parameters of the previous calculation, the integrated luminosity is maximized at $\hat{f} \approx 0.3$, with a rather broad maximum. (See Figure 3.)

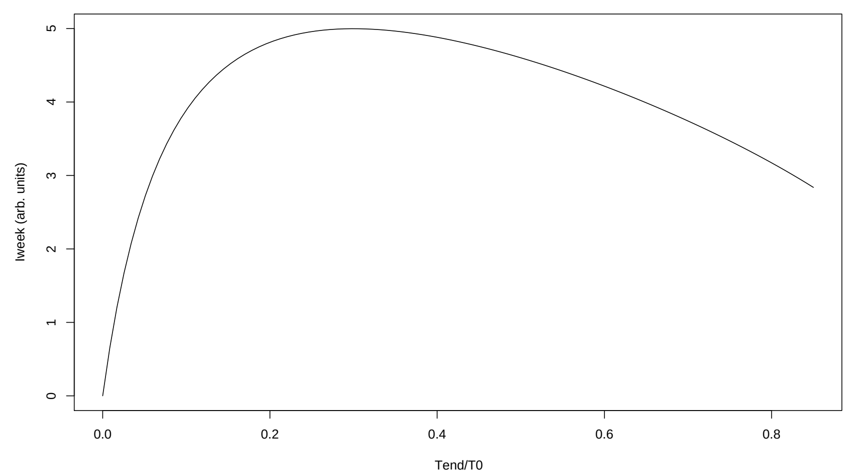

Figure 3: Weekly integrated luminosity of "perfect stores" as a function of the fraction, f, of "ultimate" luminosity.

Under these circumstances utilizing $f = 0.3$ rather than 0.85 would generate a gain in integrated luminosity of >60%. In this case, $T_f = 8$ hours, for which 70% of I_0 is integrated, and yields 16 stores per week for an optimized integrated luminosity per week of about 100 pb^{-1}.

Note that under this scenario, the antiproton source would need to be able to produce $B \cdot N_2^0 = 36 \cdot 7 \times 10^{10} = 252 \times 10^{10}$ particles in about 10 hours, which is just a tad better than the source currently can operate at its peak performance. (Actually, this is the number that makes it to "luminosity"; the source would need to have an even better rate, to account for losses along the way.) As an example, suppose the stacking rate were 20×10^{10}/hr, and presume an efficiency of 80% from produced antiprotons to antiprotons at collision. Then observe that ending the stores at a value of $f = 0.45$ (when the luminosity is reduced to about half its initial value) will produce about 300×10^{10} antiprotons between stores, or the required 250×10^{10} at collision. The weekly number for this scenario is still just under ~ 100 pb^{-1}.

Furthermore, to end a store *early* may seem odd at first, as the luminosity decays to only 65%

of its initial value before ending the store and setting up a new one. However, as indicated in Figure 4, the integrated luminosity per week is higher, akin to "topping off" in a lepton storage ring. For the antiproton-proton collider, the name of the game, as always, is reliability – keeping the store in long enough to create the required number of antiprotons, and having the required number of antiprotons "on demand." And, as stated before, these calculations do not have all the physics of the operational collider, *i.e.*, emittance growth, *etc.* Still, near-50 pb^{-1} per week, 50% of the quoted "*true* optimum" above, is quite remarkable given the numerous militating factors at work. The above arguments also suggest that as long as the antiproton source can keep up, ending stores earlier and having more frequent stores could lead to higher integrated luminosity for the physics run.

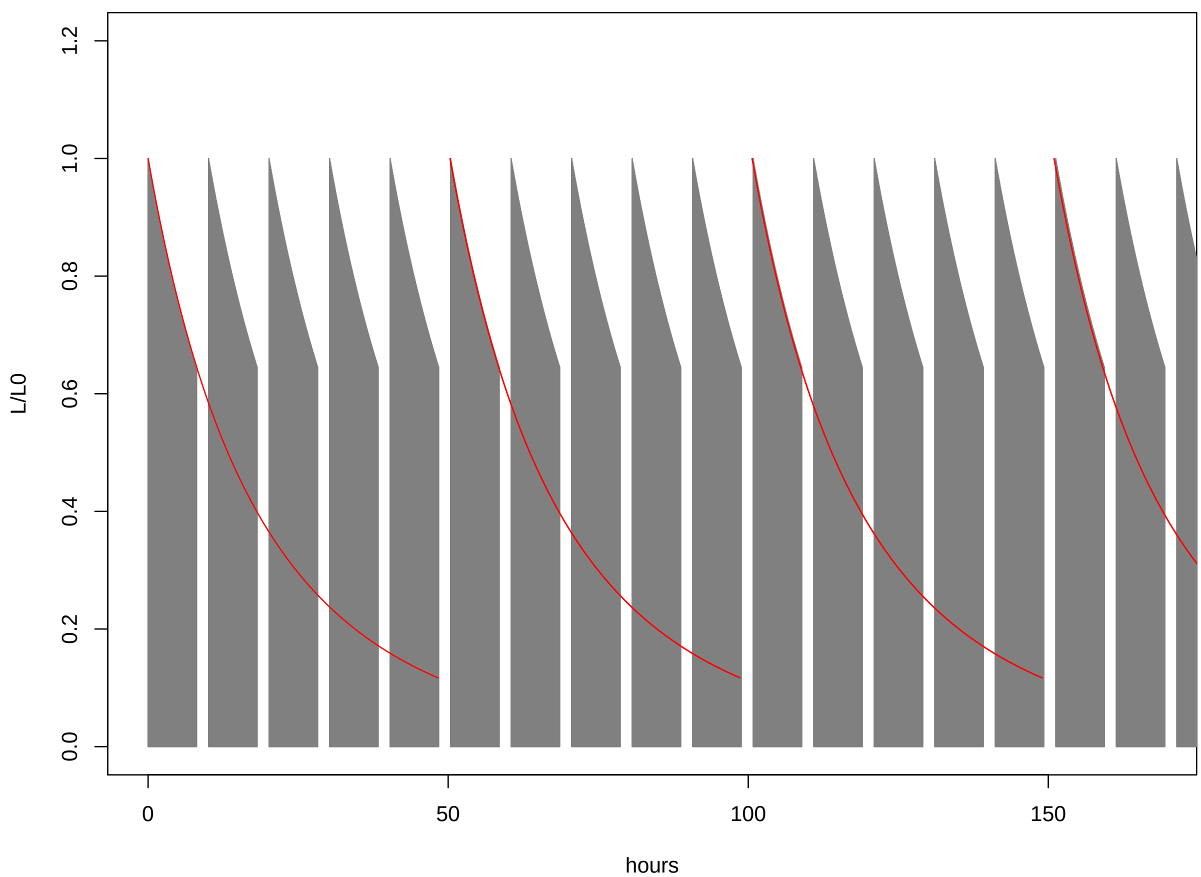

Figure 4: Shaded area is integrated luminosity throughout one week for $f = 0.3$; lines (red, in color) show development of luminosity for $f = 0.85$. So long as particles can be produced on demand, integrating over shorter times can deliver more integrated luminosity (almost twice as much) per week. See text for details.